Comment Regarding “On Delay-Independent Stability of Large-Scale Systems with Time Delays”

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Abstract—This paper comments the result of a recent paper. We point out that these results were already published in a more general framework [1], some points of which are also clarified; the state which has to be considered and a connection with the passivity theorem.

I. INTRODUCTION

In [1], the “α-stability” of a large-scale interconnected system described by functional differential equations is studied. As a matter of fact, the “asymptotic α-stability” is defined for such a system, and the usual asymptotic stability, which is considered in the paper, corresponds to the case α = 0. Roughly speaking, a system with state \( X: t \rightarrow X(t) \) is asymptotically α-stable if and only if the system with state \( t \rightarrow e^{\alpha t} X(t) \) is asymptotically stable. The connection between the result of the above-mentioned paper and the main result of [1] is pointed out in this paper. In addition, some points of [1] (such as the state which has to be considered) are also clarified. Finally, the connection between the main result of [1] and the passivity theorem is shown.

II. RESTATEMENT OF THE MAIN RESULT OF [1]

Let us first recall the basic result of [1] in the case α = 0 and clarify some points. Let \( H \) be a nonnegative real number, and let \( X_i, 1 \leq i \leq n \) be Hilbert spaces (e.g., \( X_i = \mathbb{R}^{n_i} \)); \( L_2(X_i) \) denotes the Hilbert space of functions \( x_i: [-h, \infty) \rightarrow X_i \), which are Lebesgue-measurable and square integrable on \([-h, \infty)\) (this space is equipped with the standard inner product). Set \( X = \prod_{i=1}^{n} X_i \); if \( x = (x_1, \cdots, x_n) \) and \( y = (y_1, \cdots, y_n) \) belong to \( L_2(X) \), the inner product \( (x, y) \) is defined as \( (x, y) = \sum_{i=1}^{n} (x_i, y_i) \), where \( (x_i, y_i) \) is the inner product of \( x_i \) and \( y_i \) in \( L_2(X_i) \). In all Hilbert spaces above, the norm is defined from the inner product as usually.

The truncation operator \( P_i \) to the interval \([0, t]\) is used. For any function \( x \) defined on \([-h, \infty) \), \( (P_i x)(t) = x(t) \) if \( t \in [0, t] \) and \((P_i x)(t) = 0 \) otherwise. If \( Y \) is any Hilbert space, the “extended space” \( L_2(Y) \) is the space of functions \( y: [-h, \infty) \rightarrow Y \) such that \( \forall t \in \mathbb{R}^+ \), the restriction of \( y \) to the interval \([-h, t] \) belongs to \( L_2(Y) \). For any functions \( y \) and \( z \) belonging to \( L_2(Y) \) and any \( t \in \mathbb{R}^+ \), we set \((y, z)_t = (P_y y, P_z z)\) and \( ||y||_t = ||P_y|| \) (i.e., the norm of \( P_y \) in \( L_2(Y) \)).

The system \( \Sigma \) under consideration in [1] is defined by \( n \) coupled functional differential equations

\[
\dot{x}(t) = F_i(x_i)(t) + H_i(x)(t), \quad t \geq 0 \tag{1}
\]

where \( F_i \) is a causal operator on \( L_2(X_i) \) to \( L_2(X_i) \) and \( H_i \) is a causal operator on \( L_2(X) \) to \( L_2(X) \), with \( F_i(0) = H_i(0) = 0 \).

In addition, assume that \( F_i(x_i)(t) \) and \( H_i(x)(t) \) only depend on the restrictions of \( x_i \) and \( x \), respectively, to \([t-h, t]\). The state \( X(t) \) is defined as the function \( \tau \rightarrow x(t + \tau), \tau \in [-h, 0] \), and its norm is defined by \( ||X(t)||^2 = ||x(t)||^2 + \int_{-h}^{0} ||x(t + \tau)||^2 d\tau \) (see, e.g., manuscript received July 14, 1995. The author is with Électricité de France, 92141 Clamart Cedex, France, and École Normale Supérieure de Cachan, 94230 Cachan, France (e-mail: henri.bourles@exa.edf.dgf.fr).

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IV. Conclusion

As a conclusion, note that [1, Th. 2] has been applied to the case of linear delay systems in [2]. Note also that this theorem is connected with the well-known passivity theorem when the interconnections are zero, i.e., $H_i = 0, i = 1, \ldots, n$. The $i$th subsystem $\Sigma_i$ defined by (1) can be viewed as defined by the operator $G_{ii}: z_i \rightarrow x_i$, with

$$z_i(t) = \int_0^t \gamma_i(t) \, dt,$$

and output $-x_i$; $G_{ii}$ is passive. By (4), $G_{ii}$ is strictly passive, and by (2) it is finite-gain $L_2$-stable. Hence, $z_i$ belongs to $L_2(X_i)$ (see, e.g., [7]). The proof of the global attractivity of zero is then straightforward: $\int_0^t \| \gamma_i(t+\tau) \|^2 \, d\tau$ tends to zero as $t$ tends to infinity. In addition, $\int_0^t \| \gamma_i(t+\tau) \|^2 \, d\tau$ tends to zero as $t$ tends to infinity. Hence so does $\| \gamma_i(t) \|$, i.e., zero is globally attractive for $\Sigma_i$. For proving that in addition zero is Lyapunov-stable for $\Sigma_i$, see [3] and use the definition of the state given above (at this step, it is necessary to use the fact that $\alpha_i(\| X_0 \|)$ tends to zero as $\| X_0 \|$ tends to zero). The condition ii) of the theorem means that the interconnections are weak enough for the large-scale system $\Sigma$ to remain asymptotically stable.

For a discrete-time version of this theorem in the case where the interconnections are zero, see [4] and [5]; it is connected with the small gain theorem.

REFERENCES


