$$\tilde{d}(k) = \frac{y^c(k+2)}{1+0.01d^2(k)} - y^c(k+1)$$
$$d(k) = 2d_1(k) + d_2(k)$$
$$d_1(k) = -\frac{1}{3}\left\{y^c(k+1) - y^c(k) + \sum_{i=-\infty}^{k-1}\left(\frac{1}{2}\right)^{k-i}(y^c(i+1) - y^c(i))\right\}$$
$$d_2(k) = -\frac{2}{3}\sum_{i=k+1}^{\infty} 2^{k-i}(y^c(i+1) - y^c(i)).$$

Remark 12: Reference [16] provides results combining nonlinear observers with the techniques of this paper to perform output tracking.

IV. Conclusions

We solved the output tracking problem for nonlinear discrete-time systems through stable inversion. Given a bounded time sequence for the output to track, we reduced the stable inversion problem to finding bounded solutions of a driven dynamical system (equation). We classified all bounded solutions of the equation in terms of a difference equation. We stated a result giving sufficient conditions under which the difference equation is solvable via a Picard process.

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Addendum to "W-Stability and Local Input–Output Stability Results"

Henri Bourlès

***Index Terms*—Extended space, local input–output stability, local small gain theorem, resolution Hilbet space, Sobolev space.**

I. Introduction

In the above paper,[1] a Sobolev space denoted as W was used in place of the well-known Lebesgue space L_2 for obtaining local input–output stability results, in particular a local version of the small gain theorem and of the passivity theorem. For stability studies, this space had to be "extended." It was claimed[1] that the signal space W has no classic extension. It is shown here that such an extension exists provided that the general notion of "extended resolution space" is used. This makes it possible to improve the results of Section II[1] and to clarify the relations between the extended spaces W_e, $L_{2,e}$, and $L_{\infty,e}$.

II. The Space W^n and its Extension

A. Definition of W^n

The space W^n is the set of functions x: $\mathbb{R}^+ \to \mathbb{R}^n$ such that x and its distributional-derivative $\dot{x}$ belong to L_2^n and $x(0) = 0$. [Note that when $n = 1, W^n$ is often denoted by $H_0^1(\Omega)$ in the mathematical literature, with $\Omega = (0, +\infty)$; we do not use this notation to avoid a confusion with Hardy spaces.] The norm of x in W^n is $\|x\|_W = (\|x\|_2^2 + \|\dot{x}\|_2^2)^{1/2}$, where $\|\cdot\|_2$ is the norm in L_2^n. Equipped with the inner product associated with this norm, W^n is a Hilbert space.

Note that every function in W^n is absolutely continuous [1], hence it is not necessary to add this assumption as in the paper.[1]

B. W^n Viewed as a Resolution Hilbert Space

The following *resolution of the identity* [2] $(E_T)_{T\geq 0}$ can be defined over W^n:

For any function $x \in W^n, E_T x$ is the function defined as follows: consider first the set A consisting of functions y such that

$$y(t) = x(t), \qquad 0 \leq t \leq T. \tag{1}$$

A is a closed convex subset of W^n, hence zero has a unique projection upon A; by definition, this projection is $E_T x$ and it is the function which minimizes $\|y\|_W$ among all functions y satisfying (1).

Proposition 1: $E_T x$ is the function defined by $E_T x(t) = x(t)$ for $0 \leq t \leq T$ and $E_T x(t) = x(T)e^{T-t}$ for $t > T$.

Proof: Set $y = E_T x$. This function minimizes $\int_T^\infty (\|y(t)\|^2 + \|\dot{y}(t)\|^2)\,dt$. From the Euler–Lagrange equation, y is a solution of the differential equation $\ddot{y} - y = 0$. In addition, y vanishes at infinity and is such that $y(T) = x(T)$.

Remark: If the same rationale is applied in L_2, the resolution of the identity then obtained is the usual *truncation operator* P_T. This operator cannot be used in W because $P_T x$ does not belong to W if

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[1] H. Bourlès and F. Colledani, *IEEE Trans. Automat. Contr.*, vol. 40, pp. 1102–1108, 1995.

$x(T)$ is nonzero. Based on this remark, it was erroneously claimed[1] that W^n has no usual extension.

From Proposition 1 one obtains the following result.

Proposition 2: The norm of $E_T x$ in W^n is

$$\|E_T x\|_W = \left(\|x\|_{2,T}^2 + \|\dot{x}\|_{2,T}^2 + \|x(T)\|^2\right)^{1/2} \tag{1}$$

where $\|\cdot\|_{2,T}$ denotes the seminorm in $L_{2,e}^n$ defined by $\|\cdot\|_{2,T} = \|P_T \cdot\|_2$.

Set $\|\cdot\|_{W,T} := \|E_T \cdot\|_W$ (and notice the difference with the definition of $\|\cdot\|_{2,T}$). The family $(\|\cdot\|_{W,T})_{T\geq 0}$ is a nondecreasing family of seminorms; it defines over W^n a topology $\mathcal{T}$ which is strictly finer than the one defined by the norm $\|\cdot\|_W$ [because if a sequence $(x_p)_{p\geq 0}$ of W^n is such that $\|x_p\|_W$ tends to zero, then $\|x_p\|_{W,T}$ tends to zero for every $T \geq 0$, but not conversely]. In accordance with [2], we take the following definition.

Definition 1: The extended resolution space W_e^n is the completion of $(W^n, \mathcal{T})$.

In other words, W_e^n is the set of functions x such that $E_T x$ belongs to W^n for every $T \geq 0$; W_e^n is equipped with the family of seminorms $(\|\cdot\|_{W,T})_{T\geq 0}$ and is complete for the associated topology (so that W_e^n is a Fréchet space).

III. Relations Between Various Extended Resolution Spaces

The following replaces Proposition 1.[1]

Proposition 3: Let $x \in W_e^n$. Then for every $T \geq 0$ the following inequality holds:

$$\|x\|_{\infty,T} \leq \|x\|_{W,T}. \tag{2}$$

The proof is similar to that of Proposition,[1] taking into account the new expression (1). Taking $T \to \infty$, one obtains the inclusion $W^n \subset L_\infty^n$ and the inequality

$$\|x\|_\infty \leq \|x\|_W.$$

Similarly, the following result improves Proposition 2;[1] in addition, it clarifies the relation between $L_{2,e}^n$ and W_e^n. Let $\boldsymbol{K}$ be the causal convolution operator with transfer matrix $\boldsymbol{K}(s) = (1+s)^{-1} I_n$.

Proposition 4: $\boldsymbol{K}$ is an extended resolution space-isomorphism from $L_{2,e}^n$ onto W_e^n. More specifically, $\boldsymbol{K}$ is one-to-one from $L_{2,e}^n$ onto W_e^n and $\|\boldsymbol{K}x\|_{W,T} = \|x\|_{2,T}$.

IV. Concluding Remarks

Compared with Lebesgue spaces (especially L_2), the Sobolev space W is somewhat particular in that it contains only differentiable functions (in the sense of distributions). The consequence is that, in place of the usual truncation operator P_T, the resolution of the identity which has to be used is another operator E_T. Equipped with this operator, W can be "extended" in a classic manner. As shown by Propositions 3 and 4, the extended space W_e has nice relations with the extended spaces $L_{2,e}$ and $L_{\infty,e}$. Part II of the paper should be replaced by the above approach whereas the rest of the paper can be left unchanged.

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Stable Adaptive Control for Nonlinear Multivariable Systems with a Triangular Control Structure

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***Abstract*—A stable adaptive controller is developed for a class of nonlinear multivariable systems using nonlinearly parametrized function approximators. By utilizing the system triangular property, integral-type Lyapunov functions are introduced for deriving the control structure and adaptive laws without the need of estimating the "decoupling matrix" of the multivariable nonlinear system. It is shown that stability of the adaptive closed-loop system is guaranteed, and transient performance is analytically quantified by mean square and L_∞ tracking error bound criteria.**

***Index Terms*—Adaptive control, function approximation, Lyapunov function, multivariable systems, stability.**

I. Introduction

Recent years have seen many significant developments in adaptive control for single-input/single-output (SISO) nonlinear systems. For multi-input/multi-output (MIMO) nonlinear systems, due to the couplings among various inputs and outputs, the control problem is more complex and few results are available in the literature. One of the major difficulties comes from the uncertainty in the input coupling matrix. Based on feedback linearization techniques, a variety of adaptive controllers have been proposed for linearizable systems (e.g., [1]–[6]). In these methods, for removing the couplings of system inputs, an estimate of the "decoupling matrix" is usually needed and required to be invertible during parameter adaptation. Therefore, additional efforts have to be made to avoid the possible singularity problem when calculating the inverse of the estimated decoupling matrix. For example, the projection algorithm was applied in [1] to keep the estimated parameters inside a feasible set in which the singularity problem does not occur. Although such a projection is a standard technique in linear adaptive control, it usually requires *a priori* knowledge for the feasible parameter set, and no systematic procedure is available for constructing such a set for a general plant [22]. In fact, even for SISO adaptive nonlinear control, solving the control singularity problem is far from trivial (see [12] and the references therein). For MIMO nonlinear systems, there is no effective scheme available at the present stage. As an alternative approach, a neural control design proposed in [2], [3] suggested that the initial values of neural network (NN) weights are chosen sufficiently close to the ideal values such that the inverse of the estimated decoupling matrix always exists and, therefore, an off-line training phase for NN's is necessary before the controller is put into the closed-loop system. In [6], an interesting adaptive controller was developed for multi-input parametric pure-feedback nonlinear systems. Global stability and convergence of the tracking errors were achieved for all initial parameter estimates lying in an open neighborhood of the true parameter space. For a special class of MIMO nonlinear plants, rigid robot systems, adaptive neural network control has been proposed based on energy-type Lyapunov functions in [11], [15].

As mentioned earlier, the couplings that exist among the inputs and outputs of MIMO nonlinear systems are the main difficulty in multivariable adaptive control. For systems without any input coupling, decentralized adaptive controllers were presented for a class of nonlinear systems with state interconnections [7]–[9]. Global stability and asymptotic tracking were obtained in [7], [8], provided that the system

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