The Exact Model-Matching Problem for Linear Time-Varying Systems: An Algebraic Approach

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***Abstract*—The exact model-matching problem is formulated and solved for linear time-varying systems. The condition for the existence of a proper solution, which is well known in the time-invariant case, is proven here to still be valid in the time-varying case. The properness is characterized using the Smith–MacMillan form at infinity, recently defined by the authors for the transfer matrices with time-varying coefficients.**

***Index Terms*—Linear time-varying systems, model matching, properness.**

I. Introduction

The exact model-matching problem consists of assigning the whole transfer matrix of a system. More specifically, for a given plant with proper transfer matrix $A(s)$, this problem is to find a control scheme and compensators so that the resulting system has exactly a desired transfer matrix $B(s)$ [6], [12]. If an open-loop compensator is used, the problem is to find a transfer matrix G such that

$$G(s)A(s) = B(s). \tag{1}$$

This general problem was first formulated in [21] and next studied in [11], [14], [15], and other related references for rational transfer matrices with time-invariant coefficients.

To be implementable, the solution $H(s)$ must be a *proper* transfer matrix. The necessary and sufficient condition for (1) to have a proper solution was given in terms of matrices $A(s)$ and $B(s)$ using various formalisms: in [12] and [15] minors are used to check that $A(s)$ and $F(s) \triangleq \begin{bmatrix} B(s) \\ A(s) \end{bmatrix}$ have the same *valuation* at infinity. An elegant interpretation using poles and zeros at infinity is given in [19] and [20], where it is proven that the necessary and sufficient condition for the existence of a proper solution to (1) is that A and F have the same *content* at infinity. As shown in Section III, this characterization is still valid in the time-varying case, provided that the notion of content at infinity is suitably generalized (Section II), and this is the main contribution of this note.

Though the exact model-matching problem has been widely studied for constant linear systems, this subject has never been tackled in the time-varying case. However, on one hand, the problem occurs for industrial plants with time-variant structure like flexible ac transmission systems (FACTS) (see, e.g., [13]) or when using periodic controllers for time-invariant systems to avoid an undesirable behavior of the closed loop due to unsuitable plant zeros [16]. On the other hand, in [8] it is proven, using module filtrations, that the notion of properness has the same interpretation in the time-varying case and in the time-invariant one: proper linear systems can be implemented without differentiators. For instance, for the nonproper system defined by $y = t\dot{u}$, one has to derivate once the input in order to get the output, i.e., the system has one transmission pole at infinity.

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Recent studies [3], [4] define the structure at infinity of linear time-varying systems. The approach used is based on the theory of non commutative principal ideal domains and of graded modules over such rings [7] and it is strongly connected with the module-based framework developed by Fliess (see [9], [10], and related references). The practical calculations involve only matrices [2]; particularly, the Smith–MacMillan form at infinity of a rational matrix with coefficients in a non commutative field was defined and studied. This framework is used here to formulate and solve the exact model-matching problem for linear time-varying systems. Preliminary results were given in [17].

II. Mathematical Tools

A. Noncommutative Rings and Matrices Over Such Rings: A Brief Review

The notation is that used in [4] to which the reader is referred for more details (and to [7] for basic mathematical results). Let $\mathbf{K} \supseteq \Re$ be a ground differential field, i.e., a commutative field equipped with a derivation denoted by "$\cdot$", $\mathbf{R} := \mathbf{K}[s]$ denotes the ring of polynomials with coefficients in $\mathbf{K}$ and indeterminate $s = d/dt$, and $\mathbf{R}$ is equipped with the following "commutation rule": for every a in $\mathbf{K}$

$$sa = as + \dot{a} \tag{2}$$

(Leibniz rule). In other words, $\mathbf{R}$ is the ring of differential operators with coefficients in $\mathbf{K}$, the field of (possibly) time-varying coefficients. The ring $\mathbf{R}$ is a (left and right) principal ideal domain. Its field of left fractions and its field of right fractions exist and coincide ("Ore property"); this quotient field is denoted by $\mathbf{F}$.

Set $\sigma = 1/s$ ("integration operator"). Consider the ring $\mathbf{S} := \mathbf{K}[[\sigma]]$ of formal power series in σ, i.e., consisting of elements $\mathbf{a}$ of the form

$$\mathbf{a} = \sum_{i=0}^{+\infty} a_i \sigma^i \tag{3}$$

where $a_i \in \mathbf{K}$. This ring is equipped with the commutation rule deduced from (2)

$$\sigma a = a\sigma - \sigma \dot{a} \sigma \tag{4}$$

(rule of "integration by parts"). The ring $\mathbf{S}$ has a quotient field, which is the field $\mathbf{L} := \mathbf{K}((\sigma))$ of Laurent series in σ, and $\mathbf{F}$ can be embedded in $\mathbf{L}$ (in other words, every element of $\mathbf{F}$ can be considered as an element of $\mathbf{L}$, which is of the form $\sum_{i \geq \nu} a_i \sigma^i$, $\nu \in \mathbb{Z}$, $a_\nu \neq 0$).

The "Smith form" of a matrix $A \in \mathbf{S}^{n \times m}$ exists and is $\begin{bmatrix} \operatorname{diag}\{\sigma^{\mu_i}\} & 0 \\ 0 & 0 \end{bmatrix}$, $1 \leq i \leq r$, where $0 \leq \mu_1 \leq \cdots \leq \mu_r$ (this form can be obtained using the three classic elementary column and row operations). As in the usual commutative case, the σ^{μ_i}, $1 \leq i \leq r$, are called the *invariant factors* of A [7]. The integer r is the *rank* of A over $\mathbf{S}$, i.e., the size of the largest minor with a non zero "Dieudonné determinant" [1]; r is also the rank of A over $\mathbf{L}$. The μ_i, $1 \leq i \leq r$, are called the *structural indices* of A, μ_r is called its *order*, and $\mu_1 + \cdots + \mu_r$ is called its *degree* [4].

Let $H = H(\sigma)$ be a matrix with entries in $\mathbf{L}$. The following fact generalizes the classic Smith–MacMillan form to matrices with entries in $\mathbf{L}$: there exist matrices P'_k and U, unimodular over $\mathbf{S}$, such that $P_k'^{-1} H U = \begin{bmatrix} \operatorname{diag}\{\sigma^{\nu_i}\} & 0 \\ 0 & 0 \end{bmatrix}$, $1 \leq i \leq r$, where $\nu_1 \leq \cdots \leq \nu_r$. The integers ν_i, $1 \leq i \leq r$ are uniquely defined from H, and $P_k'^{-1} H U$ is called the Smith–McMillan form of H over $\mathbf{L}$.

Consider now a matrix $G = G(s)$ with entries in $\mathbf{F}$ (i.e., the transfer matrix of a linear time-varying system [10]). As mentioned earlier, $\mathbf{F}$ can be embedded in $\mathbf{L}$, so that $G(\sigma^{-1}) = H(\sigma)$ can be considered as a matrix with entries in $\mathbf{L}$. The Smith–McMillan form of H over $\mathbf{L}$ completely describes the structure at infinity of G. The following notions are usual in the case $\mathbf{K} = \Re$ [12], [18], and have been generalized to the case of any differential ground field [4]: the integers ν_i, $1 \le i \le r$, are the *structural indexes* of $G(s)$ at infinity. If $\nu_1 < 0$, $-\nu_1$ is the *order* of the *pole* of $G(s)$ at infinity, and denoting by Σ_p the sum of all negative ν_i, $-\Sigma_p$ is the *degree* of the *pole* of $G(s)$ at infinity [written δ(pole of G at ∞)]. Similarly, if $\nu_r > 0$, this integer is the *order* of the *zero* of $G(s)$ at infinity, and denoting by Σ_z the sum of all positive ν_i, Σ_z is the *degree* of the zero of $G(s)$ at infinity [written δ(zero of G at ∞)].

Obviously, $G(s)$ can be expanded as $G(s) = \sum_{i=\nu_1}^{\infty} \Theta_i \sigma^i$ where $\Theta_{\nu_1} \neq 0$. Hence, the transfer matrix $G(s)$ is *proper* (resp., *strictly proper*) iff $\nu_1 \ge 0$ (resp., $\nu_1 \ge 1$).

B. Content at Infinity: Extension to the Time-Varying Case

As was said in Section I, the content at infinity of a transfer matrix is a key notion for solving the exact model-matching problem. The aim of this section is to generalize this notion to linear time-varying case. The following preliminary result is also needed.

Proposition 1: Let $A(s) \in \mathbf{F}^{m\times r}$ and $B(s) \in \mathbf{F}^{l\times r}$ be full-column rank[1] transfer matrices. If the pole and the zero at infinity of $F(s) \triangleq \begin{bmatrix} A(s) \\ B(s) \end{bmatrix}$ have degree zero, then the poles at infinity of A and B have degree zero. ♦

Proof: Set $\sigma = 1/s$ and let $F(1/\sigma) = \begin{bmatrix} A(1/\sigma) \\ B(1/\sigma) \end{bmatrix} = \begin{bmatrix} \underline{A}(\sigma) \\ \underline{B}(\sigma) \end{bmatrix}$. The Smith–MacMillan form at infinity of A (resp., of B) is of the form $\sigma^{-k'} P'^{-1} \underline{\underline{A}} U'$ (resp., $\sigma^{-k''} P''^{-1} \underline{\underline{B}} U''$) where k' (resp., k'') is the least common denominator (up to similarity) of all entries of $\underline{A}$ (resp., of $\underline{B}$) and $\underline{A}(\sigma) = \sigma^{-k'} \underline{\underline{A}}(\sigma)$ where $\underline{\underline{A}}$ is a matrix with entries in $\mathbf{S}$ (resp., $\underline{B}(\sigma) = \sigma^{-k''} \underline{\underline{B}}(\sigma)$ where $\underline{\underline{B}}$ is a matrix with entries in $\mathbf{S}$). P' and U' (resp., P'' and U'') are unimodular matrices over $\mathbf{S}$ which give the Smith form of $\underline{\underline{A}}$ (resp., $\underline{\underline{B}}$): $P'^{-1} \underline{\underline{A}} U' = \begin{bmatrix} \operatorname{diag}\{\sigma^{\nu_1}, \ldots, \sigma^{\nu_r}\} \\ 0 \end{bmatrix}$ (resp., $P''^{-1} \underline{\underline{B}} U'' = \begin{bmatrix} \operatorname{diag}\{\sigma^{\mu_1}, \ldots, \sigma^{\mu_r}\} \\ 0 \end{bmatrix}$) (see Section II.A, [3], and [4]). Let $k = \max\{k', k''\}$. Consider the case $k = k' > k''$. Then, the Smith–McMillan form at infinity of F is $\sigma^{-k} \begin{bmatrix} \operatorname{diag}\{\sigma^{\alpha_i}\} \\ 0 \end{bmatrix}$, where $\alpha_i = \min\{\nu_i, \mu_i + k - k''\}$, $1 \le i \le r$. As the pole and the zero at infinity of F have degree zero, it follows $\alpha_i = k$, $1 \le i \le r$. One obtains that $k = \min\{\nu_i, \mu_i + k - k''\}$, $1 \le i \le r$ from which $k \le \nu_i$ and $k'' \le \mu_i$, $1 \le i \le r$, i.e., the poles at infinity of A and B, have degree zero. The same rationale can be made in the case $k = k'' > k'$.

The following notion was introduced in [19] and [20] to study the structure at infinity of linear invariant systems, and is now generalized to the time-varying case.

Definition 1: Let $G(s)$ be a transfer matrix with entries in $\mathbf{L}$. The *content of G at infinity*, denoted by $c_\infty(G)$, is: $c_\infty(G) = \delta(\text{pole of } G \text{ at } \infty) - \delta(\text{zero of } G \text{ at } \infty)$.

The following property is the key point for solving the model-matching problem. The formulation in the time-varying case is the same as in the invariant one (see, e.g., [12] and [19]), but the proof is slightly different since the Binet–Cauchy theorem does not hold in case of non commutative fields.

[1]The property holds even when this assumption is released. To simplify the proof, we consider here only the full rank case.

Proposition 2: Let $G_1(s) \in \mathbf{F}^{l\times r}$, $G_2(s) \in \mathbf{F}^{r\times m}$ be transfer matrices of rank r. Then $c_\infty(G) = c_\infty(G_1) + c_\infty(G_2)$, where $G(s) = G_1(s)G_2(s)$. ♦

Proof: Set $\sigma = 1/s$ and let $G(1/\sigma) = G_1(1/\sigma)G_2(1/\sigma)$. Consider the Smith–MacMillan form at infinity of G_1: there exist unimodular matrices $V_1 \in \underline{\mathbf{S}}^{l\times l}$, $V_2 \in \mathbf{S}^{r\times r}$, such that $V_1^{-1}(\sigma)G_1(1/\sigma)V_2(\sigma) = \begin{bmatrix} \overline{G}_1(\sigma) \\ 0 \end{bmatrix}$, where $\overline{G}_1(\sigma) = \operatorname{diag}\{\sigma^{\mu_1}, \ldots, \sigma^{\mu_r}\}$. It follows that $c_\infty(G_1) = -\sum_{i=1}^{r} \mu_i$. Similarly, we construct the Smith–MacMillan form at infinity of G_2: there exist unimodular matrices $W_1 \in \mathbf{S}^{r\times r}$, $W_2 \in \mathbf{S}^{m\times m}$, such that $W_1^{-1}(\sigma)G_2(1/\sigma)W_2(\sigma) = [\overline{G}_2(\sigma) \quad 0]$, where $\overline{G}_2(\sigma) = \operatorname{diag}\{\sigma^{\nu_1}, \ldots, \sigma^{\nu_r}\}$. It follows that $c_\infty(G_2) = -\sum_{i=1}^{r} \nu_i$. Thus, G can now be written as $G(1/\sigma) = V_1(\sigma) \begin{bmatrix} \overline{G}_1 \\ 0 \end{bmatrix} \underbrace{V_2^{-1} W_1(\sigma)}_{U(\sigma)} [\overline{G}_2 \quad 0]\, W_2^{-1}(\sigma)$, where $U(\sigma) \in \mathbf{S}^{r\times r}$ is unimodular and $\overline{G}_1(\sigma)U(\sigma)\overline{G}_2(\sigma)$ is non singular. Let $U_1 \in \mathbf{S}^{r\times r}$, $U_2 \in \mathbf{S}^{r\times r}$ be unimodular matrices such that

$$U_1^{-1}(\sigma)\overline{G}_1(\sigma)U(\sigma)\overline{G}_2(\sigma)U_2(\sigma) = \operatorname{diag}\{\sigma^{\alpha_1}, \ldots, \sigma^{\alpha_r}\} \tag{5}$$

is the Smith–MacMillan form at infinity of $\overline{G}_1 U \overline{G}_2$. Obviously, $\begin{bmatrix} \operatorname{diag}\{\sigma^{\alpha_1}, \ldots, \sigma^{\alpha_r}\} & 0 \\ 0 & 0 \end{bmatrix}$ is the Smith–MacMillan form at infinity of G and , by taking the "Dieudonné determinant" [1] of the left- and the right-hand sides of (5), it results $\sigma^{\mu_1+\cdots+\mu_r}\sigma^{\nu_1+\cdots+\nu_r} = \sigma^{\alpha_1+\cdots+\alpha_r}$, from which the conclusion follows:

$$c_\infty(G) = -\sum_{i=1}^{r} \alpha_i = -\sum_{i=1}^{r} (\mu_i + \nu_i) = c_\infty(G_1) + c_\infty(G_2).$$

III. Exact Model Matching for Time-Varying Systems

A. Necessary and Sufficient Condition for the Existence of a Proper Solution

We are now ready to formulate and solve the exact model-matching problem in the time-varying case. The formulation is inherited from the time-invariant case [21], [11], [15].

Exact Model-Matching Problem: Given the transfer matrices $A(s) \in \mathbf{F}^{m\times r}$, $m \ge r$, $B(s) \in \mathbf{F}^{l\times r}$, find $G(s)$ which satisfies (1).♦

Theorem 1: Suppose that[2] $\operatorname{rank}(A) = \operatorname{rank}(F)$, where $F(s) \triangleq \begin{bmatrix} B(s) \\ A(s) \end{bmatrix}$. The exact model-matching problem has a proper solution iff $c_\infty(A) = c_\infty(F)$. ♦

Proof: The results settled in Section II-B allows us to conserve the same guideline as in [12], [15], and [19]: factorize F as $F(s) = \begin{bmatrix} B(s) \\ A(s) \end{bmatrix} = \underbrace{\begin{bmatrix} \overline{B}(s) \\ \overline{A}(s) \end{bmatrix}}_{\overline{F}(s)} Q(s)$ where the *pole and the zero* at infinity of $\overline{F}(s)$ have degree zero and $Q(s)$, $\overline{F}(s)$ are full rank. Then, (1) can be written as

$$G(s)\overline{A}(s) = \overline{B}(s) \tag{6}$$

[2]This condition is necessary and sufficient for (1) to have a solution. Usually it is assumed that $\boldsymbol{A}$ has full column rank (see, e.g., [6] and [12]). Here, we treat the general case; in a practical point of view, considering that $\boldsymbol{A}$ is not full column rank means that the input variables are not independent. The plant model can then be changed in order to reduce the number of physical input variables, i.e., to obtain a full-column rank transfer matrix.

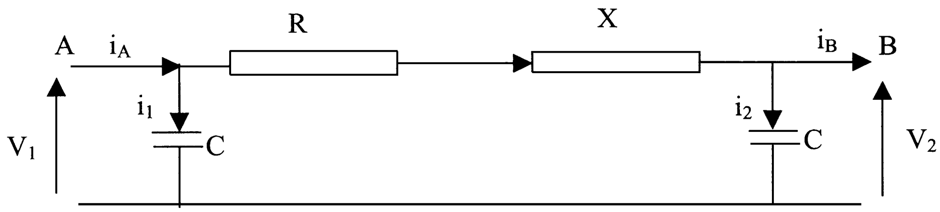

Fig. 1. EHV line mathematical model.

where $\overline{A}$ is left invertible; from (6) one obtains

$$G(s) = \overline{B}(s)\overline{A}^{-L}(s) \tag{7}$$

where $\overline{A}^{-L}(s)$ is a left inverse of $\overline{A}$.

By Proposition 1, the *pole and the zero* at infinity of $\overline{A}(s)$ and $\overline{B}(s)$ have degree zero. From (7), one deduces that the pole at infinity of G has degree zero iff the zero at infinity of $\overline{A}(s)$ has degree zero, or equivalently, iff $c_\infty(\overline{A}) = 0$. Now, $\overline{A}(s)Q(s) = A(s)$, where $\overline{A}(s)$ and $Q(s)$ are full-column rank and full-row rank, respectively. Using Proposition 2, one obtains

$$c_\infty(A) = c_\infty(\overline{A}) + c_\infty(Q). \tag{8}$$

As Q and F have the same structure at infinity, (8) yields $c_\infty(A) = c_\infty(\overline{A}) + c_\infty(F)$. As a result, G is proper iff $c_\infty(A) = c_\infty(F)$.

B. An Industrial Example

The above condition is tested for a thyristor controlled switched capacitor (TCSC). It is a FACTS often used in the power systems industry to control the power flow on a transmission line of an electrical grid [13]. Roughly speaking, it consists in a thyristor controlled reactance placed in series with the electrical line on which the power flow must be controlled. Thus, it can be modelized as a equivalent reactance between two grid nodes which continuously varies from the initial value α to the final one β

$$X(t) = \alpha - (\alpha - \beta)t. \tag{9}$$

This model is used to take into account the line parameter evolution for an upper control level of the hierarchical grid control, i.e., the electrical high voltage (EHV) control which is, for instance, the control of 225-kV and 400-kV lines in France. The mathematical model used in this case for the EHV lines is the so called π-equivalent scheme [13] given in Fig. 1.

The equations describing the circuit in Fig. 1 are

$$\begin{aligned} Cv_1 &= i_1 \\ Cv_2 &= i_2 \\ v_1 &= R(i_A - i_1) + X\frac{d}{dt}(i_A - i_1) + v_2 \\ i_A - i_1 &= i_B + i_2. \end{aligned} \tag{10}$$

It has been shown in [5] that in order to control the circuit in Fig. 1 without impulsive motions, two currents have to be chosen as inputs. Let us take $u = [\, i_A \quad i_B \,]^T$ and $y = [\, v_1 \quad v_2 \,]^T$. Let us normalize C, setting $C = 1$ and assume that the value of R is neglectible ($R = 0$). The transfer function $A(s)$ from u to y is

$$A(s) = \begin{bmatrix} D^{-1}(s)N(s) & -D^{-1}(s) \\ -D^{-1}(s)N(s) + s^{-1} & D^{-1}(s) - s^{-1} \end{bmatrix} \tag{11}$$

where $D(s) = [2 + \dot{X} + Xs]s$, $N(s) = 1 + \dot{X}s + Xs^2$ and $X(t)$ is given by (9). We wonder whether there exists a proper transfer matrix $G(s)$ satisfying (1) with $B(s)$ of the form: $B(s) = \begin{bmatrix} k_1 s^{-1} & 0 \\ 0 & k_2 s^{-1} \end{bmatrix}$. In other words, we try to find a feedforward compensator to decouple the transfer $u \mapsto y$ and to retrieve the usual time invariant relation between voltage and current as in the first two equations of (10). Set $\sigma = 1/s$ and compute first the Smith–MacMillan for at infinity of A

$$A\left(\frac{1}{\sigma}\right) = \begin{bmatrix} D^{-1}\left(\frac{1}{\sigma}\right) N\left(\frac{1}{\sigma}\right) & -D^{-1}\left(\frac{1}{\sigma}\right) \\ -D^{-1}\left(\frac{1}{\sigma}\right) N\left(\frac{1}{\sigma}\right) + \sigma & N^{-1}\left(\frac{1}{\sigma}\right) - \sigma \end{bmatrix} \tag{12}$$

where $D(1/\sigma) = (2 + \dot{X})/\sigma + X/\sigma^2$, $N(s) = 1 + \dot{X}/\sigma + X/\sigma^2$. Using the commutation rule (4), one obtains (13), as shown at the bottom of the page, with $P(\sigma) = \alpha - (\alpha - \beta)t + (2 - \beta)\sigma$. Since $P(\sigma)$ is a unit of **S**[7],it can be skipped from (13) when looking for the Smith–MacMillan form. Moreover, using the three classic elementary column and row operations [4], one can further transform (13) to finally obtain the Smith–MacMillan form $\begin{bmatrix} 1 & 0 \\ 0 & \sigma \end{bmatrix}$, hence, $c_\infty(A) = 1$. Similar calculations yield $c_\infty\left(\begin{bmatrix} B \\ A \end{bmatrix}\right) = 1$. Therefore, by Theorem 1, the exact model-matching problem admits a proper solution in this case.

IV. Concluding Remarks

The exact model-matching problem has been formulated for linear time-varying systems and a necessary and sufficient condition for the existence of a proper solution has been given. This contribution proves that the formalism introduced in [2]–[4] to study the linear time-varying systems and particularly their structure at infinity is appropriate to extend many classical topics of linear constant systems to the time-varying case. An immediate extension of the present work is to define a procedure to systematically compute the proper solution of the exact model-matching problem in the general case of multiple-input–multiple-output. In the case of large-scale systems, the calculations become rather tedious if made by hand, but are well defined and can be formalized in symbolic computing algorithms. In addition, the decoupling problem, which is closely related to the model-matching problem can be solved in the time-varying case with this formalism, as shown through the example of Section III.B.

$$A\left(\frac{1}{\sigma}\right) = \begin{bmatrix} P^{-1}(\sigma)[\alpha - (\alpha - \beta)t - \beta\sigma + \sigma^2] & -P^{-1}(\sigma) \\ P^{-1}(\sigma)[-\alpha + (\alpha - \beta)t + (\alpha + \beta - (\alpha - \beta)t)\sigma - (1 - \beta)\sigma^2 + \sigma^3] & P^{-1}(\sigma)[1 - P(\sigma)\sigma] \end{bmatrix} \tag{13}$$

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Global Adaptive Control of Nonlinearly Parametrized Systems

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***Abstract*—In this note, we consider global adaptive control of nonlinearly parametrized systems in parametric-strict-feedback form. Unlike previous results, we do not require *a priori* bounds on the unknown parameters, which is as in the linear parametrization case. We also allow unknown parameters to be time-varying provided they are bounded. Our proposed adaptive controller is a switching type controller, in which the controller parameter is tuned in a switching manner via a switching logic. Global stability results of the closed-loop system have been proved.**

***Index Terms*—Adaptive control, logic-based switching, nonlinear parameterization, nonlinear systems.**

I. Introduction

The past decade has witnessed many achievements in the design of adaptive controllers for nonlinear systems. Among them perhaps the most significant one is the development of global adaptive controllers for nonlinear systems in so-called parametric-strict-feedback (PSF) form [1] and [2]. In the original PSF form, the unknown parameters are required to enter the state equations linearly. Later, some efforts are made to remove such a linear parametrization requirement. In [3] and [4], the problem of adaptive control of nonlinearly parametrized PSF systems was first considered. However, their proposed adaptive controllers ensure only local stability and moreover, require *a priori* bounds on the unknown parameters. Recently, a new error model approach is invented to deal with nonlinear parametrization problem [5], which can be applied to develop global adaptive controllers for nonlinearly parametrized PSF systems [6]. However, their approach also requires a priori knowledge of parametric bounds. Note that an important advantage of adaptive control over robust control is that it can dispense with the need to know *a priori* bounds on the unknown parameters. Note also that for the original (i.e., linearly parametrized) PSF systems, knowing parametric bounds is not a necessary condition for developing global adaptive controllers. These two points motivate us to consider global adaptive control of nonlinearly parametrized PSF systems without a priori knowledge of parametric bounds.

This note is organized as follows. In Section II, we describe the class of nonlinear systems to be considered. In Section III, we present the adaptive control design. The stability of the closed-loop system is analyzed in Section IV and a simulation example is given in Section V. Finally, this note is concluded in Section VI.

II. Problem Formulation

In this note, we will consider global adaptive control of the following nonlinearly parametrized PSF systems:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1, \theta) \\ \dot{x}_2 = x_3 + f_2(x_1, x_2, \theta) \\ \quad \vdots \\ \dot{x}_n = u + f_n(x_1, \ldots, x_n, \theta) \end{cases} \tag{1}$$

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