Robust feedback linearization

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Abstract

Classical feedback linearization has poor robustness properties and cannot be easily combined with a H_∞ or H_2 type control law. We propose here to transform by feedback the original nonlinear system into its linear approximation around a given operating point, and prove that this allows to preserve the good robustness properties obtained by a linear control law which it is associated with. This method constitutes a way of robustly controlling an uncertain nonlinear system around an operating point.

1 Introduction

In this paper, we present a robust method for feedback linearization. We consider a nonlinear system with n states and m inputs described in a state-space form by

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i = f(x) + g(x)u \qquad (1)$$

with

$$g(x) \triangleq [\ g_1(x) \quad \cdots \quad g_m(x)\ ],\ u \triangleq [\ u_1 \quad \cdots \quad u_m\ ]^T,$$

in which $x \in \mathbb{R}^n$ denotes the state, $u \in \mathbb{R}^m$ is the control input, and $f(x)$, $g_1(x), \cdots, g_m(x)$ are smooth vector fields defined on an open subset of $\mathbb{R}^n$. We assume that the state is available for the control.

The main advantage of the classical feedback linearization method [1] comes from the fact that applying to system (1) an appropriate control law

$$u_c(x, w) = \alpha_c(x) + \beta_c(x)w,$$

the feedback system becomes linear from new input w, so it can be regulated using a classical linear feedback. Still, this method has two main drawbacks [2]. Firstly, since the system is clearly linearized by simplifying its nonlinearity, this treatment can turn out not being robust if the nonlinearity is uncertain. Moreover, in many linear control laws (like for example H_∞ control or H_2 control), the required performance is specified by weighting (frequentially or not) various variables. As a consequence, it is difficult to combine theses methods with the classical linearization, since the obtained system has no physical meaning: whatever the original nonlinear system is, one usually comes down to the same Brunovsky form (multiple chains of integrators). For example, designing a weighting function associated with u in order to take into account some saturations will be extremely difficult, because the design has to be done considering w, which is not in reality applied to the system, and the behaviour will be very different depending on whether one considers u or w, so a robust design with respect to w may be not robust at all with respect to u.

In this paper, we propose a linearization method which we believe can overcome these two difficulties. For, we use a fundamental idea of automatic control: a feedback which modifies too much the natural behaviour of a system has little chance of being robust. As a consequence, feedback linearization must perform on the original system the most little possible transformation. Looked at from that point of view, a natural step is to use the nonlinear feedback which transforms the original system into its tangent linearized system around a given operating point. Although this idea is natural, its relevance has to be proved. This is done in the sequel, using an appropriate approach for stability and robustness, namely "$\mathcal{W}$-stability" [3].

The paper will be organized as follows. In section 2, we recall some facts about $\mathcal{W}$-stability. In section 3 we propose a robust feedback linearization method, whose properties are clarified in section 4.

2 Review of $\mathcal{W}$-stability

In order to analyze the properties of the robust feedback linearizing method that will be proposed, let us firstly recall some results from [3] about $\mathcal{W}$-stability. This approach makes use of the Sobolev space W^n of functions $h : \mathbb{R}^+ \rightarrow \mathbb{R}^n$ such that h and its distributional derivative $\dot{h}$ belong to

L_2^n. The norm in W^n is then defined as

$$||h||_W = \left[\int_0^\infty h^T(t)h(t)dt + \int_0^\infty \dot{h}^T(t)\dot{h}(t)dt\right]^{\frac{1}{2}},$$

and the notion of local W-gain can be defined as usually [4].

DEFINITION 1 : Let be $\mathbf{G} : W^n \to W^m$ a time-invariant nonlinear system and

$$\begin{aligned} K &= \{k > 0,\ \exists\epsilon > 0 : ||\mathbf{G}u||_W \le k||u||_W, \\ &\qquad \forall u \in W^n \text{ such that } ||u||_W < \epsilon\}. \end{aligned}$$

If K is nonempty, $\mathbf{G}$ is said to be locally-W-stable (l-W-s) and

$$\gamma_{Wl}(\mathbf{G}) = \inf(K)$$

is called the local-W-gain of $\mathbf{G}$.

Consider now that $\mathbf{G} : W^n \to W^m$ is a time-invariant nonlinear system described by state-space equations

$$\begin{aligned} \dot{x} &= F(x, u) \\ y &= H(x, u) \end{aligned}$$

with equilibrium point $(x, u) = (0, 0)$. Let

$$G(s) = C(sI - A)^{-1}B + D$$

be the linear approximation of $\mathbf{G}$ around its equilibrium point, i.e.

$$A = \frac{\partial F}{\partial x}\Big|_{0,0},\ B = \frac{\partial F}{\partial u}\Big|_{0,0},\ C = \frac{\partial H}{\partial x}\Big|_{0,0},\ D = \frac{\partial H}{\partial u}\Big|_{0,0}.$$

The following result illustrates an important property of Sobolev space W^n.

PROPOSITION 1 : Assume that (A, B) is stabilizable, (C, A) is detectable and $G \in H_\infty$. Then $\mathbf{G}$ is l-W-s and $\gamma_{Wl}(\mathbf{G}) = ||G||_\infty$.

On this basis, one can obtain a local version of the well-known Small Gain Theorem.

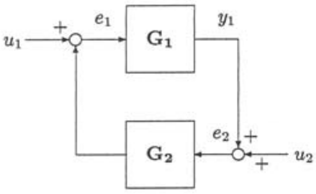

Figure 1

THEOREM 1 : Consider the standard closed-loop system in Fig.1, where $\mathbf{G_1} : W^n \to W^m$ and $\mathbf{G_2} : W^m \to W^n$. Assume that this closed-loop system is well-posed [5], i.e. there exist two operators $\mathbf{H_1}$ and $\mathbf{H_2} : W^{n+m} \to W^{n+m}$ such that

$$e = \mathbf{H_1}u \text{ and } y = \mathbf{H_2}u,$$

with

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},\ y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix},\ \text{and } u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

Then, if

$$\gamma_{Wl}(\mathbf{G_1})\gamma_{Wl}(\mathbf{G_2}) < 1,$$

the closed-loop system is l-W-s, i.e. $\mathbf{H_1}$ and $\mathbf{H_2}$ are l-W-s.

This result provides a tool to analyze in an input-output framework the robustness properties of a nonlinear closed-loop system when small moves are considered. Intuitively, completeness of the small W-gain condition will guarantee stability with respect to "small signals". Moreover, although one loses the global aspect of the Small Gain Theorem, this local version is less conservative since only local stability is concerned. This will be used in section 4 to demonstrate the robustness properties of the feedback linearization method proposed in the next section.

3 A robust method for feedback linearization

Contrarily to the classical feedback linearization which transforms the original nonlinear system (1) into a Brunovsky form, the present method consists in transforming it into its tangent linearized system around an operating point, here chosen as $x = 0$, that is

$$\dot{z} = Az + Bv \tag{2}$$

with

$$A \triangleq \frac{\partial f}{\partial x}\Big|_{x=0} \text{ and } B \triangleq g(0).$$

For, suppose that distributions $G_0, G_1, \cdots, G_{n-1}$ defined as

$$\begin{aligned} G_0 &= \text{span}\{g_1, \cdots, g_m\}, \\ G_1 &= \text{span}\{g_1, \cdots, g_m, ad_f g_1, \cdots, ad_f g_m\}, \\ &\cdots \\ G_i &= \text{span}\{ad_f^k g_j : 0 \le k \le i,\ 1 \le j \le m\}, \end{aligned}$$

for $i = 0, 1, \cdots, n-1$, satisfy the classical hypotheses

(i) distribution G_i has constant dimension near $x = 0$ for $0 \le i \le n-1$,
(ii) distribution G_{n-1} has dimension n,
(iii) distribution G_i is involutive for $0 \le i \le n-2$.

Then, as is well known [1], there exist real-valued functions $\lambda_1(x), \cdots, \lambda_m(x)$ defined on a neighborhood $\mathcal{U}$ of $x = 0$ satisfying, for numbers $r_1, \cdots, r_m$ such that $r_1 + \cdots + r_m = n$,

(j) for all $i \in [1, m]$, all $j \in [1, m]$ and all $x \in \mathcal{U}$,

$$L_{g_i}\lambda_j(x) = L_{g_i}L_f\lambda_j(x) = \cdots = L_{g_i}L_f^{r_j-2}\lambda_j(x) = 0,$$

(jj) the $m \times m$ matrix

$$M(x) \triangleq \begin{bmatrix} L_{g_1}L_f^{r_1-1}\lambda_1(x) & \cdots & L_{g_m}L_f^{r_1-1}\lambda_1(x) \\ L_{g_1}L_f^{r_2-1}\lambda_2(x) & \cdots & L_{g_m}L_f^{r_2-1}\lambda_2(x) \\ \cdots & \cdots & \cdots \\ L_{g_1}L_f^{r_m-1}\lambda_m(x) & \cdots & L_{g_m}L_f^{r_m-1}\lambda_m(x) \end{bmatrix}$$

is nonsingular at $x = 0$. We will denote $M \triangleq M(0)$.

Consider on this basis the associated classical linearizing state feedback

$$u_c(x, w) = \alpha_c(x) + \beta_c(x)w \quad (3)$$

with

$$\alpha_c(x) \triangleq -M^{-1}(x)N(x),\ \beta_c(x) \triangleq M^{-1}(x),$$
$$N(x) \triangleq \left[\begin{array}{cccc} L_f^{r_1}\lambda_1(x) & L_f^{r_2}\lambda_2(x) & \cdots & L_f^{r_m}\lambda_m(x) \end{array}\right]^T$$

and change of coordinates

$$x_c = \phi_c(x) \quad (4)$$

given by

$$\begin{array}{rcl} \phi_c(x) & \triangleq & \left[\begin{array}{ccc} \phi_{c_1}(x) & \cdots & \phi_{c_m}(x) \end{array}\right]^T \\ \phi_{c_i}(x) & \triangleq & \left[\begin{array}{cccc} \lambda_i(x) & L_f\lambda_i(x) & \cdots & L_f^{r_i-1}\lambda_i(x) \end{array}\right]^T. \end{array}$$

Then one has the following result.

THEOREM 2 : Consider system (1) and suppose that $f(x)$ and $g(x)$ satisfy hypotheses (i), (ii) and (iii). Then, under state feedback

$$u(x, v) = \alpha(x) + \beta(x)v$$

and change of coordinates

$$z = \phi(x)$$

defined by

$$\begin{array}{rcl} \alpha(x) & \triangleq & \alpha_c(x) + \beta_c(x)LT^{-1}\phi_c(x), \\ \beta(x) & \triangleq & \beta_c(x)R^{-1}, \\ \phi(x) & \triangleq & T^{-1}\phi_c(x), \end{array} \quad (5)$$

where

$$L \triangleq -M \cdot \frac{\partial \alpha_c}{\partial x}\Big|_{x=0},\ T \triangleq \frac{\partial \phi_c}{\partial x}\Big|_{x=0} \text{ and } R \triangleq M^{-1},$$

system (1) is transformed into system (2).

Proof: Applying classical linearizing feedback (3) together with change of coordinates (4), it is known from [1] that original nonlinear system (1) is transformed into

$$\dot{x_c} = A_c x_c + B_c w \quad (6)$$

where

$$A_c = \left[\begin{array}{ccccc} A_{c_1} & 0_{r_1\times r_2} & 0_{r_1\times r_3} & \cdots & 0_{r_1\times r_m} \\ 0_{r_2\times r_1} & A_{c_2} & 0_{r_2\times r_3} & \cdots & 0_{r_2\times r_m} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0_{r_m\times r_1} & 0_{r_m\times r_2} & 0_{r_m\times r_3} & \cdots & A_{c_m} \end{array}\right],$$

each $r_i \times r_i$ matrix A_{c_i} being equal to

$$A_{c_i} = \left[\begin{array}{ccccc} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{array}\right],$$

and

$$B_c = \left[\begin{array}{ccccc} B_{c_1} & 0_{r_1\times 1} & 0_{r_1\times 1} & \cdots & 0_{r_1\times 1} \\ O_{r_2\times 1} & B_{c_2} & 0_{r_2\times 1} & \cdots & 0_{r_2\times 1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0_{r_m\times 1} & 0_{r_m\times 1} & 0_{r_m\times 1} & \cdots & B_{c_m} \end{array}\right],$$

each $r_i \times 1$ matrix B_{c_i} being equal to

$$B_{c_i} = \left[\begin{array}{ccccc} 0 & 0 & \cdots & 0 & 1 \end{array}\right]^T.$$

On the other hand, (A, B) being a controllable pair, it is well known from classical linear control theory (see [6] for example) that there exist matrices T $(n \times n)$, L $(m \times n)$ and R $(m \times m)$ such that

$$T(A - BRL)T^{-1} = A_c \text{ and } TBR = B_c,$$

T and R being non singular.

On this basis, it is immediate to see that system (6) is transformed into (2) using feedback

$$w = LT^{-1}x_c + R^{-1}v \quad (7)$$

and change of coordinates

$$z = T^{-1}x_c. \quad (8)$$

Combining (3),(4) with (7),(8), it is clear that feedback

$$\begin{array}{rcl} u & = & \alpha_c(x) + \beta_c(x)w \\ & = & \alpha_c(x) + \beta_c(x)LT^{-1}\phi_c(x) + \beta_c(x)R^{-1}v \\ & \triangleq & \alpha(x) + \beta(x)v \end{array} \quad (9)$$

together with change of coordinates

$$z = \phi(x) = T^{-1}\phi_c(x) \quad (10)$$

transforms original nonlinear system (1) into its tangent linearized version (2) around $x = 0$.

Moreover, linear parts of (9) and (10) obviously satisfy

$$\frac{\partial \alpha}{\partial x}\Big|_{x=0} = 0,\ \frac{\partial \phi}{\partial x}\Big|_{x=0} = I_{n\times n} \text{ and } \beta(0) = I_{m\times m},$$

since these feedback and change of coordinates have no influence on the linear part of original nonlinear system (1). As a consequence, one has

$$T = \frac{\partial \phi_c}{\partial x}\Big|_{x=0},\ L = -M \cdot \frac{\partial \alpha_c}{\partial x}\Big|_{x=0} \text{ and } R = M^{-1},$$

which concludes the proof. △

4 Robustness analysis of the method

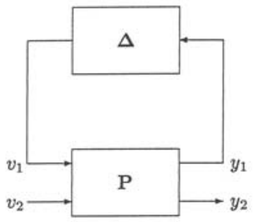

Figure 2

As was said above, we claim that this method has certain robustness properties which can be analyzed using the concept of $\mathcal{W}$-stability. Consider in Fig.2, as in [7], the standard diagram for robustness analysis of an uncertain system, where $y_2 \in \mathbb{R}^{p_2}$ represents the measured signals, in our context supposed to be state $x \in \mathbb{R}^n$ of nonlinear time-invariant system $\mathbf{P}$ (i.e. $p_2 = n$), $v_2 \in \mathbb{R}^{m_2}$ is the control signal, and $y_1 \in \mathbb{R}^{p_1}$, $v_1 \in \mathbb{R}^{m_1}$ are introduced to depict the uncertainty $\mathbf{\Delta}$ operating on the original system. $\mathbf{\Delta}$ is nonlinear, time-invariant, and such that

$$\gamma_{W_i}(\mathbf{\Delta}) \leq \delta.$$

This system can be represented in a state-space form by

$$\begin{array}{rcl} \dot{x} & = & f(x) + g_1(x)v_1 + g_2(x)v_2 \\ y_1 & = & h(x) + k_1(x)v_1 + k_2(x)v_2 \\ y_2 & = & x \end{array} \quad . \qquad (11)$$

Suppose that we are interested in robustly controlling this uncertain system around a given operating point $x = x^*$, chosen here for convenience as $x^* = 0$. A possible way to deal with this problem makes use of the linearizing feedback law derived in section 2.

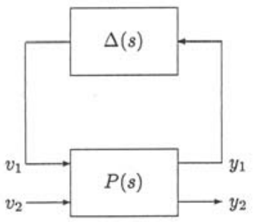

Figure 3

For, let us consider in Fig.3 the corresponding linear problem, where $P(s)$ is the transfer function of the linear approximation of $\mathbf{P}$ around $x = 0$, i.e.

$$P(s) \; : \; \begin{array}{rcl} \dot{x} & = & Ax + B_1 v_1 + B_2 v_2 \\ y_1 & = & Cx + D_1 v_1 + D_2 v_2 \\ y_2 & = & x \end{array} \qquad (12)$$

with

$$A = \frac{\partial f}{\partial x}\Big|_{x=0}, \; C = \frac{\partial h}{\partial x}\Big|_{x=0},$$

$$B_1 = g_1(0), \; B_2 = g_2(0), \; D_1 = k_1(0), \text{ and } D_2 = k_2(0),$$

Suppose that one has found a state feedback linear controller K (no matter how it has been constructed) which stabilizes $P(s)$ and such that the transfer function from v_1 to y_1, i.e. $\mathcal{F}_l(P, K)$, satisfies

$$||\mathcal{F}_l(P(s), K)||_\infty < \frac{1}{\delta}.$$

If this is the case, the closed-loop system in Fig.4 is stable for all uncertainty Δ_l satisfying $||\Delta(s)||_\infty \leq \delta$.

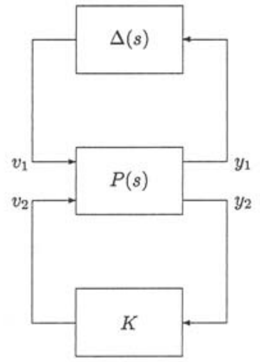

Figure 4

Returning now to the uncertain nonlinear system of Fig.2, let us associate the linearizing feedback of Theorem 2

$$\mathbf{LF} \; : \; \begin{array}{rcl} v_2 & = & \alpha(x) + \beta(x)v_2' \\ z & = & \phi(x) \end{array}$$

where $\alpha(x)$, $\beta(x)$ and $\phi(x)$ are constructed according to Theorem 2 on the basis of $f(x)$ and $g_2(x)$, to linear controller K, that is

$$v_2' = Kz.$$

The system relying v_2' to $y_2' = z$ is now linear. However, considering the general closed-loop system in Fig.5, $\mathbf{\Lambda} : (v_1, v_2') \rightarrow (y_1, y_2')$ is still nonlinear and given by

$$\begin{array}{rcl} \dot{z} & = & Az + B_2 v_2' + \left[\frac{\partial \phi}{\partial x} \cdot g_1(x)\right]_{x=\phi^{-1}(z)} \cdot v_1 \\ y_1 & = & [h(x) + k_2(x)\alpha(x)]_{x=\phi^{-1}(z)} + [k_1(x)]_{x=\phi^{-1}(z)} \cdot v_1 \\ & & + [k_2(x)\beta(x)]_{x=\phi^{-1}(z)} \cdot v_2' \\ y_2' & = & z. \end{array} \qquad (13)$$

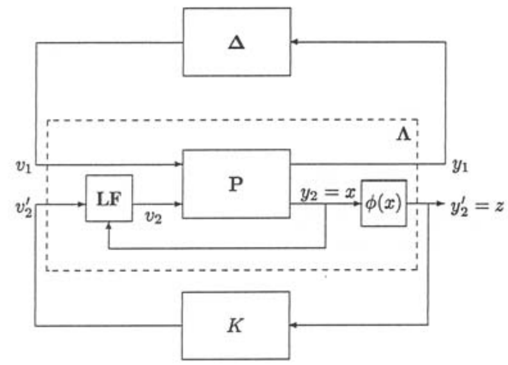

Figure 5

A crucial issue concerns the point at which one evaluates robustness (see [8]). Indeed, one is interested in obtaining robustness for system $\mathbf{P} : (v_1, v_2) \to (y_1, y_2)$ controlled by K and $\mathbf{LF}$ (Fig. 6.1), but linear controller K, which has been designed to this aim (at least around $x = 0$ since it has been computed on the basis of $P(s)$), is now acting on $\mathbf{\Lambda}$ (Fig.6.2), and this situation can deteriorate robustness properties that have been obtained. However, the key point is that $\mathbf{\Lambda}$ and $\mathbf{P}$ have in fact the same linear approximation around the operating point $x = 0$, thanks to the particular linearizing feedback used here.

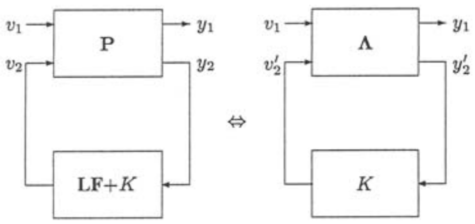

Figure 6.1 Figure 6.2

Let us define as $\mathbf{T}(\mathbf{\Lambda}, K)$ the nonlinear operator relying v_1 to y_1 in Fig.6.2. According to state-space equations (13) of $\mathbf{\Lambda}$, $\mathbf{T}(\mathbf{\Lambda}, K)$ is given by

$$\begin{aligned} \dot{z} &= (A + B_2 K) z + \left[\frac{\partial \phi}{\partial x} \cdot g_1(x) \right]_{x=\phi^{-1}(z)} \cdot v_1 \\ y_1 &= [h(x) + k_2(x)\alpha(x)]_{x=\phi^{-1}(z)} + [k_1(x)]_{x=\phi^{-1}(z)} \cdot v_1 \\ &\quad + [k_2(x)\beta(x)]_{x=\phi^{-1}(z)} \cdot K \cdot z. \end{aligned} \tag{14}$$

Then one has the following result.

THEOREM 3: Consider the closed-loop system in Fig.5 where $\mathbf{P}$ is given by equations (11). Suppose that $f(x)$ and $g_2(x)$ satisfy the hypotheses of Theorem 2, and that linearizing feedback $\mathbf{LF}$ and change of coordinates $\phi(x)$ are given by (5). If linear controller K stabilizes $P(s)$ defined in (12) and satisfies

$$||\mathcal{F}_l(P(s), K)||_\infty < \frac{1}{\delta}, \tag{15}$$

then $\mathbf{T}(\mathbf{\Lambda}, K)$ is locally W-stable and

$$\gamma_{Wl}(\mathbf{T}(\mathbf{\Lambda}, K)) < \frac{1}{\delta}.$$

As a consequence, the present feedback ensures local-W-stability for every nonlinear uncertainties $\mathbf{\Delta}$ such that $\gamma_{Wl}(\mathbf{\Delta}) \leq \delta$.

Proof: Firstly, Let $T(s)$ be the linear approximation of $\mathbf{T}(\mathbf{\Lambda}, K)$ around $z = 0$. Considering state equations (14) with $A + B_2 K$ stable, applying Proposition 1 leads to

$$\gamma_{W_l}(\mathbf{T}(\mathbf{\Lambda}, K)) = ||T(s)||_\infty.$$

Then, since clearly

$$T(s) = \mathcal{F}_l(\Lambda(s), K),$$

where $\Lambda(s)$ is the linear approximation of $\mathbf{\Lambda}$ around $x = 0$, one has

$$\gamma_{W_l}(\mathbf{T}(\mathbf{\Lambda}, K)) = ||\mathcal{F}_l(\Lambda(s), K)||_\infty. \tag{16}$$

On the other hand, $\Lambda(s)$ is defined as

$$\Lambda(s) \ : \ \begin{array}{rcl} \dot{z} &=& \bar{A}x + \bar{B}_1 v_1 + \bar{B}_2 v'_2 \\ y_1 &=& \bar{C}x + \bar{D}_1 v_1 + \bar{D}_2 v'_2 \\ y'_2 &=& z \end{array},$$

the different matrices being computed on the basis of (13). Because of

$$\left.\frac{\partial \alpha}{\partial x}\right|_{x=0} = 0, \ \left.\frac{\partial \phi}{\partial x}\right|_{x=0} = \left.\frac{\partial \phi^{-1}}{\partial z}\right|_{z=0} = I_{n \times n} \text{ and } \beta(0) = I_{m \times m},$$

one has

$$\bar{A} = A, \ \bar{B}_1 = \left[\frac{\partial \phi}{\partial x} \cdot g_1(x) \right]_{x=\phi^{-1}(0)} = B_1, \ \bar{B}_2 = B_2,$$

$$\begin{aligned} \bar{C} &= \left[\frac{\partial h}{\partial x} + \frac{\partial k_2}{\partial x} \cdot \alpha(x) + k_2(x) \cdot \frac{\partial \alpha}{\partial x} \right]_{x=\phi^{-1}(0)} \\ &\quad \cdot \ \left.\frac{\partial \phi^{-1}}{\partial z}\right|_{z=0} \\ &= C, \end{aligned}$$

$$\bar{D}_1 = D_1 \text{ and } \bar{D}_2 = [k_2(x)\beta(x)]_{x=\phi^{-1}(0)} = D_2,$$

thus proving that $\Lambda(s) = P(s)$. Then, for any linear controller K achieving (15), one has, remembering (16),

$$\gamma_{W_l}(\mathbf{T}(\mathbf{\Lambda}, K)) = ||\mathcal{F}_l(P(s), K)||_\infty < \frac{1}{\delta}.$$

As a consequence, according to Theorem 1, the closed-loop interconnection is l-W-stable for every nonlinear uncertainties such that $\gamma_{Wl}(\mathbf{\Delta}) \leq \delta$. $\triangle$

Remark 2: This method, which confers to the feedback linearized system the behaviour of the original nonlinear system for small moves around $x = 0$ (here supposed to be the design point), will lose its robustness properties for systems having very strong nonlinearities, since the linear behaviour imposed by the linearized feedback law will be too constraining. Still, as far as nonlinear robust control around an equilibrium point is concerned, consider that the linear controller K achieves the optimal value of attenuation (which is not often desirable, but is here supposed for clarity of discussion), i.e.

$$||\mathcal{F}_l(P(s), K)||_\infty < \frac{1}{\delta_{max}},$$

then the linearizing feedback proposed here is the best one that can be associated with this linear controller, since it does not deteriorate the closed-loop system robustness properties (and clearly no linearizing feedback can improve them). Moreover, it can ensure in any case stability for an exact model all over the operating domain, and as a consequence is clearly better than the linear controller alone (this is shown in [10] in the particular case of a steam turbine).

Remark 3: There exist an infinite number of pairs (linear controller-linearizing feedback) achieving the same result. In particular, there exists a linear controller K_c to be associated with the classical linearizing feedback, i.e.

$$K_c = (R^{-1}K - L)T^{-1},$$

but practically, this is not a convenient way for designing controllers, since a physical analysis is required in H_2 or H_∞ methods.

Remark 4: The method described here for robustly controlling uncertain systems must be compared with nonlinear H_∞ control ([9],[7]). A theoretical comparison is not easy. Still, a possible difference could concern the size of the signals considered, since $\mathcal{W}$-stability (see Definition 1) allows to analyse only small moves of a nonlinear system. However, the proposed method does not involve the numerical problems of nonlinear H_∞ control associated with Hamilton-Jacobi equations or NLMI's.

Remark 5: We have given in this paper a general theoretical framework to ideas that had been applied with success on the particular example of a steam turbine ([2],[10]).

Remark 6: The method developed here for state feedback linearizable systems can be applied in an input-output linearization context to asymptotically minimum phase nonlinear systems, but the details need to be worked out.

5 Conclusion

In this paper we have proposed a method for robustly controlling around a design point a nonlinear feedback linearizable system $\mathbf{P}$. This method consists in associating a robust linear control law to a particular linearizing feedback. This feedback, instead of resulting in the classical Brunovsky form, is computed in order that the feedback linearized system coincides with $P(s)$, the tangent linearized system of $\mathbf{P}$ around this design point. Using then $\mathcal{W}$-stability, it is proved that this is the only feedback that does not deteriorate the robustness properties obtained by the linear control law.

References

[1] A. Isidori, *Nonlinear control systems*, Springer-Verlag, Heidelberg, 1989.

[2] H. Bourlès, F. Colledani and M.P. Houry, "Robust continuous speed governor control for small-signal and transient stability", *IEEE Trans. Power Syst.*, vol. PS-12, pp. 129-135, 1997.

[3] H. Bourlès and F. Colledani, "$\mathcal{W}$-stability and local input-output stability results", *IEEE Trans. Automat. Contr.*, vol. AC-40, pp. 1102-1108, 1995.

[4] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*, New York: Academic, 1975.

[5] J. C. Willems, *The Analysis of Feedback Systems*, Cambridge, MA: MIT Press, 1971.

[6] T. Kailath, *Linear systems*, Prentice Hall, Englewood Cliffs, N.J., 1980.

[7] W.-M. Lu and J.C. Doyle, "Robustness analysis and synthesis for nonlinear uncertain systems", *IEEE Trans. Automat. Contr.*, **42**, pp. 1654-1662, 1997.

[8] J. C. Doyle and G. Stein, "Robustness with observers", *IEEE Trans. Automat. Contr.*, **24**, pp. 607-611, 1979.

[9] A. Isidori and A. Astolfi, "Disturbance attenuation and H_∞ control via measurement feedback in nonlinear systems", *IEEE Trans. Automat. Contr.*, **37**, pp. 1283-1293, 1992.

[10] F. Colledani, *Régulation coordonnée tension/vitesse des groupes turbo-alternateurs*, PHd thesis, Paris-Sud Orsay, 1995.